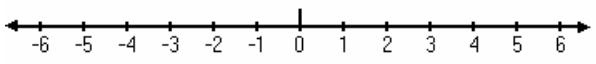
Signed Number Stuff

Here are some notes on stuff I've done on signed numbers. I used this stuff with kids in grades 4-6 who test a couple of grade levels ahead. The material below was used before we got into algebra with positive and negative numbers.

There are lots of word problems that might be useful in upper el, or at least give you some ideas for coming up with problems that will match your kids.

Greater than and less than on the number line



$3 < 5$	Is 3 to the left or right of 5 on the number line?
$-3 < 6$	Is -3 to the left or right of 6 on the number line?
$-6 < 3$	Is -6 to the left or right of 3 on the number line?
$0 < 2$	Is 0 to the left or right of 2 on the number line?
$-1 < 0$	Is -1 to the left or right of 0 on the number line?
$-6 < -3$	Is -6 to the left or right of -3 on the number line?

In general, if $a < b$, then what do you know about the positions of a and b on the number line?

Now look at these:

$5 > 3$	Is 5 to the left or right of 3 on the number line?
$6 > -3$	Is 6 to the left or right of -3 on the number line?
$3 > -6$	Is 3 to the left or right of -6 on the number line?
$10 > 0$	Is 10 to the left or right of 0 on the number line?
$0 > -1$	Is 0 to the left or right of -1 on the number line?
$-3 > -6$	Is -3 to the left or right of -6 on the number line?

If $a > b$, what do you know about the positions of a and b on the number line?

Differences in altitude

Try solving these two problems:

The top of Mt. Tremblant is 3,150 feet above sea level. The top of Mt. Jacques Cartier is 1,277 feet above sea level. How many feet higher is the top of Mt. Tremblant than the top of Mt. Jacques Cartier? Show your work.

The lowest point of the St. Lawrence River is 294 feet below sea level. The top of Mt. Jacques Cartier is 1,277 feet above sea level. How many feet higher is the top of Mt. Jacques Cartier than the lowest point of the St. Lawrence River? Show your work.

How are these two problems different? How do you write out these problems?

Fill out this table:

$3 - 2 = 1$
$3 - 1 = 2$
$3 - 0 = 3$
$3 - (-1) = ?$
$3 - (-2) = ?$

Now try this problem:

Sam built dog house. He didn't have a lot of wood, so he built the dog house over a hole one foot deep to give his dog extra room. The roof of the dog house is three feet above ground level. How much room does the dog have from the top of the dog house to the bottom?

Changing temperatures

Monday morning, it was 25 degrees outside. By the afternoon, it was 32 degrees. What was the change in temperature?

Tuesday afternoon, it was 25 degrees in the afternoon. Overnight, the temperature reached 7 degrees. What was the change in temperature?

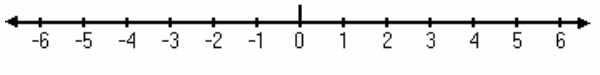
What if the temperature changes from 5 degrees below zero to 10 degrees above zero? What is the total change? Answer for the changes below. Be sure to indicate whether the temperature change is rise in temperature or a fall in temperature, along with the amount of rise or fall.

10 degrees below to 5 degrees below
10 degrees below to 12 degrees below
10 degrees below to 10 degrees above
4 degrees above to 25 degrees above

Wednesday morning, it was 5 degrees below zero in the morning. In the afternoon, the temperature rose 10 degrees. What was the temperature in the afternoon?

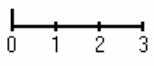
Write equations to solve the following:

The temperature was 15 degrees and then fell by 4 degrees
The temperature was 15 degrees and then rose by 4 degrees
The temperature was 15 degrees below and then rose by 20 degrees
The temperature was 15 degrees and then fell by 4 degrees
The temperature was 15 degrees and then fell by 15 degrees
The temperature was 15 degrees below and then fell by 4 degrees

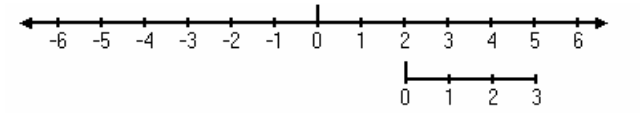


Adding a positive number

To add a positive number, move right on the number line by the amount you want to add. For example, if you want to add three to a number, we can make a piece of a number line that is three units long:

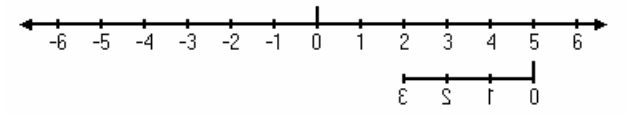


Now, we can add 3 to any number by putting the 0 of this piece over that number and reading the number that's under the three. For example, the picture below shows that 2 + 3 = 5.



Subtracting a positive number

Subtracting a positive is the reverse of adding a positive. You move left on the number line by the given amount. To subtract three from a number, go left three units on the number line. In the picture below, we show 5 – 3 = 2. Notice, to subtract, we reverse the piece of the number line that stands for positive 3:

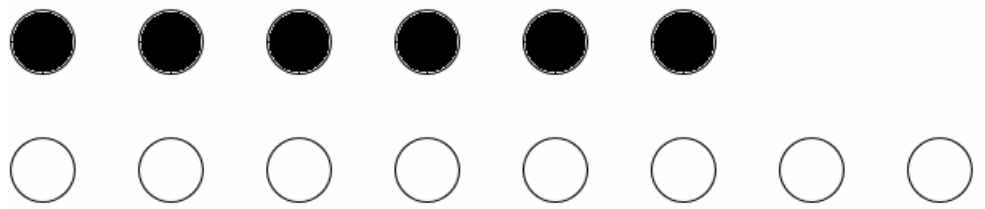


Adding a negative number

We have experience adding and subtracting positive numbers in everyday life. For example, we add positive numbers when we add up our change, or subtract when we make change. But what does it mean to add a negative number?

We can see one example of addition of negative numbers by looking at atoms. Atoms are made up of particles called protons, electrons, and neutrons. The protons have a positive charge. The electrons have a negative charge. The neutrons have zero charge. If the atom has the same number of protons and electrons, the atom is neutral, i.e., it has a zero charge.

What if an atom has 6 protons and 8 electrons? What would the total charge of the atom be? In the picture below, the protons are black and the electrons are white.



Make two atoms that are different from this one but have the same charge.

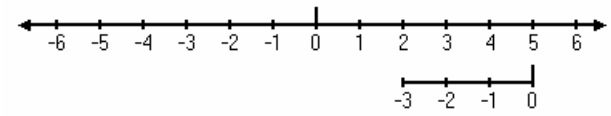
Another way to think about adding a negative is to look for patterns in different addition problems. For example, fill in the following table:

4 + 2 = 6
4 + 1 = 5
4 + 0 = 4
4 + (-1) = ?
4 + (-2) = ?

Now let's see how we use the number line to add negative numbers:

3 + 2 = 5 [move 2 units to right of 3]
3 + 1 = 4 [move one unit to the right of 3]
3 + 0 = 3 [don't move]
3 + (-1) = 2 [move one unit to the left of 3]
3 + (-2) = 1 [move 2 units to the left of 3]

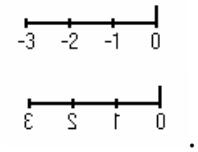
In the picture below, we show 5 + (-3). What direction do we move on the number line when we add a negative number?



Compare these equations:

5 + (-3) = 2 and 5 – 3 = 2
4 + (-3) = 1 and 4 – 3 = 1
3 + (-3) = 0 and 3 – 3 = 0
2 + (-3) = -1 and 2 – 3 = -1

So adding a negative is the same subtracting a positive. This makes sense when you compare the negative segement to the reversed positive segment:



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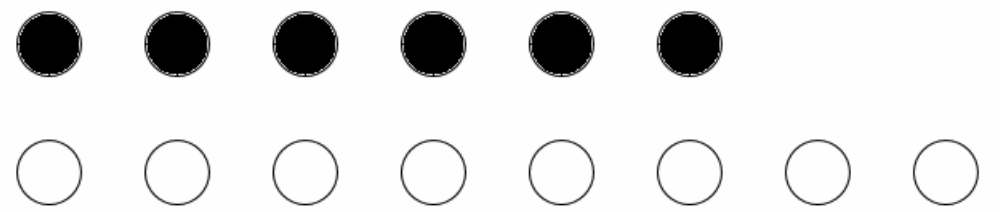
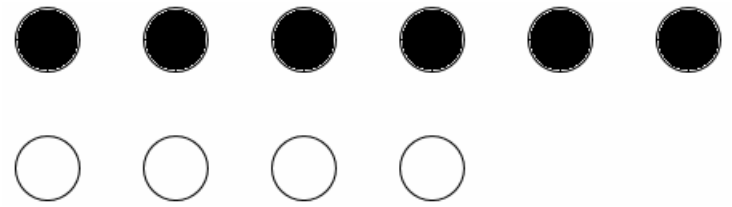
Both segments have zero on the right.

Subtracting a negative

What does it mean to subtract a negative? Again, we can go back to our example with atoms.



Remember that the protons are black and the electrons are white. We saw that the total charge of this atom is -2. What if we took away 4 electrons (4 negatively charged particles)?

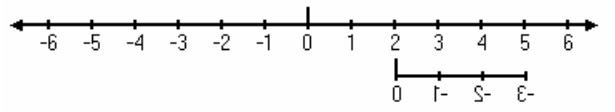


Then it would have 6 protons and 4 electrons. So the total charge would be a positive charge of 2 (Why?). This means that -2 – (-4) = ?

Remember the table:

3 – 2 = 1 [move two units to the left of 3]
3 – 1 = 2 [move one unit to the left of 3]
3 – 0 = 3 [don't move]
3 – (-1) = 4 [move one unit to the right of 3]
3 – (-2) = 5 [move 2 units to the right 3]

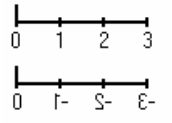
Subtracting a negative on the number line is the reverse of adding a negative. The picture below shows a number line representation of 2 – (-3):



Compare these equations:

2 + 3 = 5 and 2 – (-3) = 5
1 + 3 = 4 and 1 – (-3) = 4
0 + 3 = 3 and 0 – (-3) = 3
-1 + 3 = 2 and -1 – (-3) = 2

Adding a positive number is the same subtracting the negative of that number. This makes sense when you compare the positive segement to the reversed negative segment:



They both have zero on the left.

Exercises

-7 + 3 =

5 – (-4) =

5 – 9 =

-3 – 9 =

-3 – 6 =

-8 – (-2) =

6 + (-2) =

7 – (-5) =

-10 + 4 =

4 + (-7) =

The Mountaineer

A mountaineer began ascending a mountain at a rate of 25 feet per hour. Will he be higher up the mountain or lower down the mountain in 3 hours? By how much?

A mountaineer is ascending a mountain at a constant rate of 25 feet per hour. Was he higher up the mountain or lower down the mountain 3 hours ago? By how much?

A mountaineer is descending a mountain at a constant rate of 25 feet per hour. Will he be higher up the mountain or lower down the mountain in 3 hours? By how much?

A mountaineer is descending a mountain at a constant rate of 25 feet per hour. Was he higher up the mountain or lower down the mountain 3 hours ago? By how much?

Multiplication patterns

In an earlier section, we looked at patterns in equations to understanding addition and subtraction of negative numbers. We can do the same thing with multiplication:

$3(2) = 6$
$3(1) = 3$
$3(0) = 0$
$3(-1) = ?$
$3(-2) = ?$

Another way to look at these multiplication problems is to see them as repeated addition of groups of protons, neutrons or electrons:

$3(2) = 2 + 2 + 2 = 6$	[3 groups of protons with 2 in each group]
$3(1) = 1 + 1 + 1 = 3$	[3 groups of protons with 1 in each group]
$3(0) = 0 + 0 + 0 = 0$	[3 groups of neutrons with 0 in each group]
$3(-1) = -1 + (-1) + (-1) = ?$	[3 groups of electrons with 1 in each group]
$3(-2) = -2 + (-2) + (-2) = ?$	[3 groups of electrons with 2 in each group]

The commutative property of multiplication says that we can change the order of multiplicands and we will still get the same product. Fill out the table below:

$3(2) = 2(3) = 6$
$3(1) = 1(3) = 3$
$3(0) = 0(3) = 0$
$3(-1) = -1(3) = ?$
$3(-2) = -2(3) = ?$

Now fill out this table:

$(-3)(2) = -6$
$(-3)(1) = -3$
$(-3)(0) = 0$
$(-3)(-1) = ?$
$(-3)(-2) = ?$

Another look at the mountaineer problems

There are many cases when we want to associate a direction with a number. For example, temperature is measured in degrees above or below zero. Distance may be measured in miles east or west from a particular place. Altitude can be measured in feet above or below sea level or some other level. Time can be measured in years BC or AD. We can also talk about time in terms of "hours from now" or "hours ago".

In the mountaineer problems, we can think of ascent (going up the mountain) as movement in a positive direction and descent (going down the mountain) as movement in a negative direction. We can also think of "3 hours from now" as a positive movement through time and "3 hours ago" as a negative movement through time.

We can now write expressions with positive and negative numbers for each problem.

A mountaineer began ascending a mountain at a rate of 25 feet per hour. Will he be higher up the mountain or lower down the mountain in 3 hours? By how much?

The mountaineer is travelling in a positive direction (up the mountain) and we are moving through time in a positive direction (3 hours from now), so the expression to solve this problem is:

$(25)(3) = 75$

Since 75 is a positive number, the mountaineer will be 75 feet higher.

Exercise

Write expressions for each of the other mountaineer problems using positive and negative numbers. Solve each expression and then give the answer to the problem.

A mountaineer is ascending a mountain at a constant rate of 25 feet per hour. Was he higher up the mountain or lower down the mountain 3 hours ago? By how much?

A mountaineer is descending a mountain at a constant rate of 25 feet per hour. Will he be higher up the mountain or lower down the mountain in 3 hours? By how much?

A mountaineer is descending a mountain at a constant rate of 25 feet per hour. Was he higher up the mountain or lower down the mountain 3 hours ago? By how much?